

XX. *M. De Luc's Rule for measuring Heights by the Barometer, reduced to the English Measure of Length, and adapted to Fahrenheit's Thermometer, and other Scales of Heat, and reduced to a more convenient Expression. By the Astronomer Royal.*

Redde, Jan. 13, 1774. **M.** DE LUC, F. R. S. in a large and valuable Treatise upon the Barometer and Thermometer, lately published at Geneva, the result of many years labour and study, has given a rule for the measurement of heights by the barometer, deduced from his experiments, and far more accurate, than any published before; since it appears that he could determine heights by it generally to 10 or 15 feet, and that the error seldom, if ever, amounted to double that quantity. This valuable degree of exactness he has obtained principally by detecting the faults of the common barometer, and, in consequence, improving the construction of it; and by introducing the use of the mercurial thermometer, to accompany that of the barometer. The principal faults, which he found in the common barometers, arose from
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the repulsion of the quicksilver by the glass tube, from air and moisture admitted into the tube, and from the variations of the density of quicksilver by heat and cold; another very considerable error arose, in calculating heights from the barometer, by not allowing for the changes of the density of the air, whose gravity affords us this measure of heights, owing to heat and cold. The first cause of error, that of the repulsion of the tubes, he remedied, by substituting a syphon-barometer instead of the simple upright tube, the repulsion of the two legs of the syphon, counteracting itself; the error arising from air and moisture in the tube he cured by boiling the quicksilver, after it was put into the tube, and other precautions; the errors, in the estimation of the heights, arising from the changes of the density of the quicksilver, and density of the air by heat and cold, he shews how to correct by allowances depending on two thermometers, one attached to the frame of the barometer itself, and the other made to be exposed to the open air, to shew its degree of heat; which thermometers are to be noted both at the top and bottom of the hill. Lastly, by a great number of experiments made with accurate barometers and thermometers of his own construction, he has deduced a rule for calculating heights of places; the exactness of which he has sufficiently proved by a large table of experiments. But this rule is expressed in French measure, and is adapted either to a thermometer, whose freezing point is 0, and that of boiling water 80, or to thermometers of particular scales. It may

may be therefore useful, to reduce M. DE LUC'S rule to English measure, and to adapt it to the thermometer of Fahrenheit's scale, which is generally used in this country.

M. DE LUC, in the winter season, heated the air of his room to as great degree as he could, and noted the rise of the barometer, owing to the diminution of its density, or specific gravity, by heat; he also noted the height of the thermometer, both before and after the room was heated. Hence he deduced a rule, that when the barometer is at 27 French inches, which was the case in this experiment, an increase of heat, from freezing to that of boiling water, will raise the barometer 6 lines, or $\frac{1}{34}$ th part of the whole. It is easy to see, that when the barometer is higher than 27 French inches, this variation will increase in the same proportion; or will be always $\frac{1}{34}$ th of the height of the barometer; therefore, if the height of the barometer be called B, the rise of the barometer, for an increase of heat from freezing to boiling water, will be $\frac{B}{54}$; and, as it will be less for a less difference of heat, therefore, if the number of degrees, marked on the thermometer, between freezing and boiling water, be called K, and the rise of the thermometer from any given point be called H, the correspondent rise of the barometer will be $\frac{B}{54} \times \frac{H}{K}$, by the increase of heat from the given point by the number of degrees H. If the heat, instead of increasing, was to decrease, then H would

would signify so many degrees decrease of heat, and the barometer would sink by $\frac{B}{54} \times \frac{H}{K}$. The first temperature of heat, to which M. DE LUC thought best to reduce his observations of the barometer, is $\frac{1}{8}$ th of the interval from freezing to boiling water above the former point: and if the thermometer was higher than this degree, he subtracted $\frac{B}{54} \times \frac{H}{K}$; if it was lower, he added it to the observed height of the barometer; and thus he obtained the exact height of the barometer, such as it would have been, if the density of its quicksilver had been the same as answers to the first degree of temperature. He thus corrected the height of both his barometers (that at the bottom, and that at the top of the hill) for the particular degree of heat, indicated by a thermometer attached to the barometer, at each station; for it might and would commonly happen, that the degree of heat would be different at the two stations. The heights of the barometers, thus corrected, were what he made use of in his subsequent calculations. Calling these two altitudes of the barometer B and *b*, putting log. B and log. *b*, for the logarithms of B, and *b*, taking only the four first places of figures, after the characteristic, or considering the remaining figures as decimals, and putting C for the mean height of a thermometer, exposed to the air at top and bottom of the hill, the freezing point being 0, and the point of boiling water at 80, he finds, by his experiments, that the height of the hill will be given in French toises, when C is $16\frac{3}{4}$, by

simply taking the difference of the logarithms of the heights of the barometer, or will be equal to $\log. B - \log. b$; and in any other degree of heat, will be greater or less, in proportion as the rarity of the air is greater or less, than in the fixt temperature; or greater or less by $\frac{1}{215}$ th part of the whole, for every degree of the thermometer reckoned from the fixt temperature $16\frac{3}{4}$; and consequently the height of the place will be expressed generally in French toises, by this *formula*

$$\log. B - \log. b + \frac{\log. B - \log. b}{\frac{C - 16\frac{3}{4}}{215}}$$

$$= \log. B - \log. b \times 1 + \frac{C - 16\frac{3}{4}}{215}. \text{ To reduce this}$$

formula to English measure, and to the scale of Fahrenheit's thermometer, we should first premise some particulars. The French foot is to the English foot, as 1,06575 to 1, as was found by a very accurate experiment: see Phil. Trans. Vol. LVIII. for 1768, p. 326; and it is well known, that the point of freezing, on Fahrenheit's thermometer, is at 32, and that of boiling water at 212, or the interval between them 180 degrees. But M. DE LUC's point of boiling water 80, was marked when the barometer was at 27 French inches; and it is the custom of our principal English workmen to mark the point of boiling water, 212, on Fahrenheit's thermometer, when the barometer stands at 30 inches, which is equal to 28 inches, 1,8 lines French measure; or 13,8 lines higher than M. DE LUC's barometer, when he set off the point of boiling water on his thermometers; and it is well known, that the heat of boiling water varies with the weight of the atmosphere: M. DE LUC finds, by his experiments,

ments, this rule, that an increase of 1 line in the height of the barometer, raises the quicksilver of the thermometer, placed in boiling water, by $\frac{1}{1134}$ th part of the interval between the freezing point and that of boiling water: he afterwards indeed found, that this rule would not answer for such large variations of the barometer, as take place in ascending to very great heights above the earth's surface; *vid. Essai sur les variations du chal: de l'eau bouill:* but it is accurate enough for any small variation of the barometer, on one side or other of its mean height in these lowest regions of the atmosphere. The change therefore of the boiling point on Fahrenheit's scale, for a change of 1 line in the barometer, will be $\frac{1 \cdot 8^\circ}{1134} = 0,16$; therefore 13,8 lines will cause $0,16 \times 13,8 = 2,2$ degrees of Fahrenheit's scale; and a thermometer, whose point of boiling water was marked 212, when the barometer stood at 30 English inches = 28 inches, 1,8 lines French measure, will, when the barometer descends to 27 French inches, sink 2,2 degrees in boiling water, or to 209,8, or in round numbers to 210 degrees, which is distant only 178 from 32, the point of freezing. Hence an extent of 80° of M. DE LUC's thermometer answers to an extent of 178 of our Fahrenheit's thermometer; and putting F for the degrees of this thermometer, corresponding to C of M. DE LUC's, we shall have $C : F - 32 :: 80 : 178$, and $C = \frac{F - 32 \times \frac{80}{178}}{1}$ which substituted in M. DE LUC's formula, gives

$$\begin{aligned} & \log. \overline{B} - \log. b \times 1 + \frac{C - 16\frac{1}{2}}{215} = \\ & = \log. \overline{B} - \log. b \times 1 + \frac{F - 32 \times \frac{80}{178} - 16\frac{1}{2}}{215} = \\ & \qquad \qquad \qquad \text{Y 2} \qquad \qquad \qquad = \end{aligned}$$

$$\begin{aligned}
 &= \overline{\log. B - \log. b} \times 1 + \frac{80}{178 \times 215} \overline{F - 32 - \frac{1.75}{10} 16,75} \\
 &= \overline{\log. B - \log. b} \times 1 + \frac{F - 32 - 37,27}{478,38} = \\
 &= \overline{\log. B - \log. b} \times 1 + \frac{F - 69,27}{478,38}. \text{ Where the} \\
 &\text{answer will still come out in French toises, though} \\
 &\text{adapted to Fahrenheit's thermometer. To bring it} \\
 &\text{out in English fathom (or measure of 6 feet) multi-} \\
 &\text{ply the above expression by } 1,06575, \text{ and we shall} \\
 &\text{have } \overline{\log. B - \log. b} \times 1 + \frac{F - 69,27}{478,38} \times 1,06575 \\
 &= \overline{\log. B - \log. b} \times 409,11 + F \times \frac{1,06575}{478,38} = \\
 &= \overline{\log. B - \log. b} \times \frac{409,11 + F}{448,87}, \text{ or in round numbers} \\
 &= \overline{\log. B - \log. b} \times \frac{409 + F}{449} = \\
 &= \overline{\log. B - \log. b} \times 1 + \frac{F - 40}{449}; \text{ which will} \\
 &\text{express the height between the two stations in} \\
 &\text{English fathom.}
 \end{aligned}$$

In the foregoing expressions, B and b , as has been mentioned before, signify heights of the barometer, at the lower and higher stations, both corrected according to Mr. DE LUC's directions, for the difference of heat between a fixed temperature, (namely $\frac{1}{4}$ th of the interval between freezing and boiling water), and the present heat, indicated by the thermometer attached to the barometer at each station; but it is not necessary, to correct *both* barometers for the effect of heat, but only one for the difference of heat of the two; which will be more convenient also on another account, because the difference of heat, at the two stations, will

will be generally small, and the correction to reduce one barometer to the heat of the other will consequently be small also; whereas the difference of the present heat, and the fixt temperature, and consequently the correction of both barometers, may be frequently very considerable: this is evident: because if the heat of the barometers, at both stations, was the same, however different from the fixt temperature chosen by M. DE LUC, no correction would be necessary; the mercury in the barometer in both stations, being expanded in the same proportion, and consequently the difference of the logarithms of its height, at both stations, being the same, as if the heat of both barometers had agreed with that of the fixt temperature. I shall now therefore suppose the upper barometer is to be corrected, to reduce it to the temperature of the lower one, and that b signifies the height of this barometer, as observed, and not yet corrected; the correction, from what has been said above, calling D the difference of height of the thermometer attached to the barometer at the two stations, will be

$\pm \frac{D b}{54 K}$, according as the thermometer stands highest at the lower or upper station; and the upper barometer corrected, instead of b , will be $b \pm \frac{D b}{54 K}$, which substituted in the *formula*, gives

$\log. B - \log. \left(b \pm \frac{D b}{54 K} \right) \times 1 + \frac{F - 40}{449}$. But the correction, on account of the difference of heat of the barometer at the two stations, may be reduced to a still easier expression, in which the variable quantity b , the height of the upper barometer, shall

shall not appear. The fluxion of a logarithm is to the fluxion of its natural number, as the modulus of the system to the natural number; and 4343 is the modulus of the common logarithms, when the four places, next following the characteristic, are taken as whole numbers, instead of decimals, which is meant to be done in the use of the foregoing *formula*.

Therefore $\frac{D b}{54 K}$ being very small with respect to b , we shall have, variation of $\log. b$: variation of b ($= \frac{D b}{54 K}$) :: 4343 : b very nearly, and thence va-

riation of $\log. b = \pm \frac{D b}{54 K} \times \frac{4343}{b} = \pm \frac{4343 D}{54 K}$.

Which (putting $K = 178$) $= \pm 0,452 D$. Hence

$\log. (b \pm \frac{D b}{54 K}) = \log. b \pm 0,452 D$; which being substituted in the *formula* above, will give the difference of height, of the two stations, in English fathom, in a more convenient expression,

namely $\log. B - \log. b \mp 0,452 D \times 1 + \frac{F - 40}{449}$;

where the upper sign, —, is to be used, when the thermometer of the barometer is highest at the lower station, and the lower sign, +, is to be used, when the said thermometer is lowest at the lower station. The first case will be most common; especially where the difference of height of the two stations is considerable. It should also be observed, that when F , the height of Fahrenheit's thermometer, is less than 40° , $+\frac{F-40}{449}$ becoming negative or subtractive, must be applied in the calculation accordingly.

It may perhaps be convenient to repeat here the meaning of the algebraic terms, used in the foregoing *formula*, that any person may make use of it, without having occasion to recur to the foregoing investigation. B signifies the observed altitude of the barometer at the lower station, and b that at the upper station; $\text{Log. } B$ and $\text{Log. } b$, signify their logarithms taken out of the common tables, by assuming the four first figures next following the characteristic as whole numbers, and considering the three remaining figures, to the right hand, as decimals; D signifies the difference of height of Fahrenheit's thermometer, attached to the barometer at the top and bottom of the hill; and F signifies the mean of the two heights of Fahrenheit's thermometer, exposed freely for a few minutes to the open air in the shade, at the top and bottom of the hill.

The *formula*, for the measure of heights, may also be changed, and adapted to thermometers of particular scales, for the convenience of calculation, as M. DE LUC has done; but these scales will be different from his. The thermometer, attached to the barometer, had better be divided with the interval between freezing and boiling water, consisting of $81,4$ degrees ($=180 \times ,452$) the freezing point may be marked 0 , and the point of boiling water will be $81,4$; for then, if the difference of height of this thermometer, at the two stations, be called d , we shall have $d = ,0,452 \times D$. for $d : D :: 81,4 : 180 :: ,0,452 : 1$, and the number of degrees express'd by d , will shew immediately the correction for the difference of heat of the two barometers. If the thermometer, designed to shew the temperature of the air, be divided

divided with the interval between freezing and boiling water = 200, and the freezing point be marked — 9, and the boiling point + 191, and the heights of this thermometer, at the two stations, be called G and I, we shall have $\frac{F-40}{449} = \frac{G+I}{2 \times 500}$
 $= \frac{G+I}{1000}$. For $F-40 = F-32-8$, is the height of Fahrenheit's thermometer, reckoned from 8 degrees above freezing, and $449 : 500 :: 180 : 200 :: 8 : 9$, and the fraction $\frac{F-32-8}{449}$, if both the numerator and denominator be increased in the ratio of 449 to 500, will become $= \frac{F-32-8 \times \frac{500}{449}}{500}$
 $= \frac{F-32 \frac{500}{449} - 9}{500} = \frac{G+I}{2 \times 500} = \frac{G+I}{1000}$, because $\frac{G+I}{2} + 9$
 $= F-32 \times \frac{500}{449}$. Therefore, if the thermometer of the barometer has the freezing point marked 0, and the point of boiling water 81,4, and the difference of its height, at the two stations, be called d ; and the thermometer for measuring the temperature of the air, be divided with the interval of 200 between the freezing point and that of boiling water, and the first be marked — 9, and the latter + 191, and the degrees, shewn by this, at the two stations, be called G and I; the *formula*, that will give the height of the upper station above the lower one, in English fathoms, will be $\log. B - \log. b \mp d \times 1 + \frac{G+I}{1000}$; which consequently multiplied by 6, will give the height in English feet. It is to be observed, as before, that — d or + d is to be used, according as the thermometer, attached to the barometer, is highest at

at the lower or upper station; and if G and I should happen to fall below 0 of the scale, or to be subtractive, they must be applied accordingly in the calculation.

I shall now add nothing more, but to give the rule for finding heights by the barometer, according to the *formulæ* delivered above, in common language; first, as adapted to Fahrenheit's thermometer, and next, as adapted to the two thermometers of particular scales. Take the difference of the tabular logarithms of the observed heights of the barometer, at the two stations, considering the 4 first figures, exclusive of the index, as whole numbers, and the three remaining figures to the right as decimals, and subtract or add $\frac{4.32}{1000}$ th of the difference of the altitude of the Fahrenheit's thermometer, attached to the barometer at the two stations, according as it was highest at the lower or upper station; thus you will have the height of the upper station above the lower, in English fathom, nearly; to be corrected, as follows: make this proportion; as 449 is to the difference of the mean altitude of Fahrenheit's thermometer, exposed to the air at the two stations, from 40° , so is the height of the upper station found nearly, to the correction of the same; which added or subtracted, according as the mean altitude of Fahrenheit's thermometer was higher or lower than 40° , will give the true height of the upper station above the lower, in English fathoms; and multiplied by 6, will give it in English feet.

The same rule, adapted to the thermometers of particular scales, is this:

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Take the difference of the tabular logarithms of the observed heights of the barometer, at the two stations, considering the 4 first figures (exclusive of the index) as whole numbers, and the three remaining figures to the right as decimals; and subtract or add the difference of the thermometer, of a particular scale, attached to the barometer, at the two stations, according as it was highest at the lower or upper station, and you will have the height of the upper station above the lower one, in English fathom, nearly; to be corrected as follows: make this proportion; as 1000 is to the sum of the altitudes of the thermometer of a particular scale, exposed to the air at both stations, so is the height of the upper station above the lower, found nearly, to the correction of the same; which added or subtracted, according as the sum of the altitudes of the thermometers, exposed to the air, is positive or negative, will give the true height of the upper station above the lower in English fathoms; and multiplied by 6, will give it in English feet.

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